Treatment of Data: Chemistry and Mathematics

OBJECTIVE: To review some of the prerequisite mathematical concepts (listed below) which are important in chemistry: The first three, A-C, are numerical manipulations. The rest have to do with units and measurement, including the metric system. You must become familiar with this type of unit conversion if you are to do well in chemistry or any other science class.

A. Exponential notation
B. Significant figures
C. Rounding
D. Dimensional analysis
E. Metric system, including metric-English conversions
F. Temperature

You should refer to your textbook and talk to your instructor for more details on those subjects that are not clear to you.

A. EXPONENTIAL (SCIENTIFIC) NOTATION:

Exponential notation is a way to express large or small numbers in a short form. To express a number in exponential notation, the decimal point is moved so that there is only one digit to the left of the decimal point. That number is multiplied times 10 to the power equal to the number of places that the decimal was moved. If the decimal was moved to the left, as for a large number, the exponent is positive. If the decimal was moved to the right, as for a small number, the exponent is negative. Examples are given in Table 1.

$$C \times 10^{e}$$

for numbers >1, $e > 0$
for numbers <1, $e < 0$

between 1 and 10

Table 1. Examples of Exponential Notation

Number	Number Expressed in Exponential Notation
1,000,000	1×10^{6}
96,500	9.65×10^{4}
-9,000	-9×10^{3}
454	4.54×10^{2}
1	10^{0}
0.0100	1.00×10^{-2}
0.0000005	5×10^{-7}

Multiplication and Division Using Exponential Notation:

Exponential notation simplifies multiplication and division. Multiplying 5 by 4 is easy, but 5,000 times 4,000,000 is more cumbersome unless you use exponential notation:

Example 1. 5,000 x 4,000,000 = 20,000,000,000

Example 2. $(5 \times 10^3)(4 \times 10^6) = 20 \times 10^9 = 2 \times 10^{10}$

The operation has been split into two parts:

5 x 4 = 20
and
$(10^3)(10^6) = 10^9$

When multiplying powers of ten, the exponents are added. The result can now be expressed as the product of the parts, 20×10^9, but in exponential notation, **the decimal point must be placed after the first digit.** The product is correctly expressed as 2×10^{10}.

When dividing, the power of ten in the denominator is subtracted from the power of ten in the numerator:

Example 3. $$\frac{(0.00084)(350)}{(0.0000042)} = \frac{(8.4 \times 10^{-4})(3.5 \times 10^{2})}{4.2 \times 10^{-6}} = 7.0 \times 10^4$$

To summarize, when multiplying, the exponents are added.

$$(10^a)(10^b) = 10^{a+b}$$

When dividing, the exponents are subtracted.

$$\frac{(10^a)}{(10^b)} = 10^{a-b}$$

B. SIGNIFICANT FIGURES

There is some uncertainty in every measured quantity. Every reported result should reflect the precision of the measurement. Significant figures are important in lab because they tell us how precisely we know a quantity and how well we can reproduce a laboratory measurement. (The term "accuracy" is used to refer to how closely a value measured in the laboratory is to the "true" value.) Consider two densities, 0.8 g/mL and 0.826 g/mL. The 0.826g/mL number is more precise. The last digit on the right always indicates the "uncertain" or "doubtful" digit.

Here are the rules for determining the number of significant figures:
a. Nonzero digits are always significant.
b. Zeroes may or may not be significant, depending on where they are in the number.
   i. zeroes between other nonzero digits are always significant
   ii. zeroes to the left of the first nonzero digit are never significant
   iii. zeroes to the right of the last nonzero digit *in a decimal* are significant
   iv. zeroes to the right of the last nonzero digit, not in a decimal, may or may not be significant; use exponential notation for a precise expression of sig. figs.

Table 2. Examples of How to Determine the Number of Significant Figures

Quantity	Number of Sig. Figs.	Comments
2.00 g	3	Zeros which follow a digit and a decimal place always count.
1.0622 g	5	A zero between nonzero digits is significant.
751 students	infinite	Definite quantity, a counted number; it's exact
0.006110 cm	4	Zeroes to the left of the first nonzero digit are not significant. The last zero counts because it follows a digit.
6.110×10^{-3}	4	The last zero counts because it follows digit.
1.2×10^{8}	2	
$683,462.02	8	
7,685,000 people	4?	The last 3 zeros may or may not be significant. We know there are at least 4 sig figs; there may be more.
7.6850×10^{6} people	5	Exponential notation makes the number of significant figures explicit.
1 g = 1000 mg	Infinite	This is a definition.

Calculations with significant figures and decimal places:

Remember that significant figures and decimal places are not the same. There are two rules when doing calculations with significant figures and decimal places.

1) When multiplying and dividing, keep the same number of significant figures as in the least precise item of data; that is, the one with the fewest number of significant figures.
2) When adding and subtracting, keep the same number of decimal places as there are in the number with the fewest decimal places.

Example 4. Calculate the volume of a rectangular wooden board from the following measurements: length = 121.20 cm; width = 3.31 cm; thickness (or height) = 0.019 cm.

ANSWER: volume = (length) (width)(thickness)
volume = (121.20 cm) (3.31 cm) (0.019 cm) = 7.6226 cm^3 = 7.6 cm^3

Example 5.

$$\begin{array}{r} 18.68 \\ -\ 11.4 \\ \hline 7.28 \end{array} = 7.3$$

In this case the answer is reported to one decimal place, which is two significant figures. The limit is due to the one decimal place in 11.4. If we don't know whether this number is 11.35 to 11.45, we don't know whether the result is 7.33 to 7.23. *Note that the number of significant figures is decreased by this subtraction.*

C. ROUNDING

Keep all digits until the end of a calculation, then round to the correct number of significant figures; if the first numeral(s) beyond the number of sig figs is >5-9, round up; if the first numeral is 0-4, round down.

Rounded to two significant figures: 16.43 becomes 16; 16.6 becomes 17
16.49 becomes 16; 17.5 becomes 18

D. DIMENSIONAL ANALYSIS

The units of a number are as important as the number itself. Expressing a mass of 200 is meaningless. It could be 200 micrograms or 200 kilograms. For a number to make sense, proper units must be attached. Dimensional analysis is a technique for working with numbers that have units (dimensions). It is a method that uses units and conversion factors to solve problems.

Example 6. Grandma gives you a \$10 bill to buy apples for apple pies. A typical apple weighs 0.42 pounds, and you can buy apples for 79 cents per pound. A pie takes 6.0 apples. How many apple pies can Grandma make from her \$10?

Set up the equation first using just units. If the units cancel, the problem will be solved correctly. If the units do not cancel, the conversion factors must not be set up correctly.

$$\cancel{\$} \times \frac{\cancel{\text{cents}}}{\cancel{\text{dollar}}} \times \frac{\cancel{\text{lb.}}\ \text{apples}}{\cancel{\text{cents}}} \times \frac{\cancel{\text{apple}}}{\cancel{\text{lb.}}} \times \frac{\text{pie}}{\cancel{\text{apples}}} = \text{pies}$$

Now put in the numbers.

$$\$10 \times \frac{100\text{ cents}}{1\text{ dollar}} \times \frac{1\text{ lb. apples}}{79\text{ cents}} \times \frac{1\text{ apple}}{0.42\text{ lb.}} \times \frac{1\text{ pie}}{6.0\text{ apples}} = 5.0\text{ pies}$$

Example 7. The volume of an object is equal to the length times the width times the height of the object. What are the units for volume if length, width, and height are in units of cm?

$$V = (L)(W)(H)$$
$$V = (cm)(cm)(cm)$$
$$V = cm^3$$

Notice that dimensional analysis works even if there are no numbers with the units.

E. INTERNATIONAL SYSTEM OF UNITS AND THE METRIC SYSTEM

The international system of units, abbreviated SI, is based on the metric system. The metric system uses length, mass, and volume units which are related. For pure water at 4°C:

1.00 gram = 1.00 cubic centimeters = 1.00 milliliters, abbreviated 1.00 g = 1.00 cm^3 = 1.00 mL

The most frequently used metric units are given in Table 2 along with their English system counterparts. You should use these conversion factors for all problems in this lab manual.

Table 2. Metric and English Unit Conversions

Property Measured	Metric Unit	English Unit	Conversion Factor
length	meter (m)	inch	2.54 cm/in exactly
mass	gram (g)	pound	454 g/lb (3 sig. figs.)
volume	liter (L)	quart	1 liter/ 1.06 qt. (3 sig. figs.)

The metric system is further simplified by the use of prefixes which alter the units by powers of ten. The most common prefixes are listed in Table 3.

Table 3. Metric Prefixes

Prefix	Factor	Symbol
kilo	1×10^{3}	k
deci	1×10^{-1}	d
centi	1×10^{-2}	c
milli	1×10^{-3}	m
micro	1×10^{-6}	μ (mu)
nano	1×10^{-9}	n

Example 8: 1.0 milliliter (mL) equals 1.0×10^{-3} L

Example 9: There are 2.540 cm/in.

Example 10: 2,400 g equals 2.4 kg (to 2 significant figures).

The following examples illustrate conversions between the English and the metric system. **You must know the conversion factors within the English system and those within the metric system.**

Example 12. Convert 245 pounds to kilograms.

$$(245 \text{ pounds}) \times \frac{(453.6\ g)}{(lb)} x \frac{(kg)}{(1000\ g)} = 111 \text{ kg}$$

F. TEMPERATURE

Common Temperature Scales: Fahrenheit, Celsius, and Kelvin. In science, the Celsius and Kelvin scales are used. The relationships between the different scales are given in Table 4.

Table 4. Common Temperature Scales

Name of Scale	Symbol	Freezing Point of Water	Boiling Point of Water	Conversion* Factor
Fahrenheit	F	32 °F	212 °F	$°F = \frac{9}{5}\ °C + 32$
Celsius	C	0 °C	100 °C	$°C = \frac{5}{9}(°F - 32)$
Kelvin	K	273 K	373 K	K = °C + 273

* $\frac{9}{5}$ and 32 are exact numbers

Example 13. Convert 72 °F to °C

$$\frac{5}{9}(72 - 32) = \frac{5}{9}(40) = 22\ °C$$

Name ____

Experiment 1: LABORATORY EXERCISES—REPORT FORM

No work shown = no credit given. Use correct dimensional analysis notation as illustrated in the examples. Always use the correct number of significant figures, and be sure to write units with your answers. Use only the conversions found in the table below.

Length	**Volume**	**Mass**	**Time**
1 m = 1.0936 yd	1 qt = 4 cups	1 lb = 16 oz	1 hr = 60 min
1 in = 2.54 cm (exact)	1 qt = 2 pints	1 kg = 2.2046 lb	1 min = 60 sec
1 mi = 5280 ft	1 gal = 4 qt	1 ton = 2000 lb	1 day = 24 hr
1 Å = 1 x 10^{-10} m	1 L = 1.0567 qt		1 year = 365.25 days
12 in = 1 foot (exact)			

Convert the following.

1. 7.2 quarts to liters

ANSWER:________________________________L

2. 1.64 feet to meters

ANSWER:________________________________m

3. 760 millimeters to inches

ANSWER:________________________________in

4. 685 nm to cm

ANSWER:________________________________cm

5. 7.25 °F to °C

ANSWER:______________________________ °C

6. 38.6 °C to °F

ANSWER:______________________________ °F

7. 88.6 °F to K

ANSWER:______________________________K

8. 20 lb. to Mg

ANSWER:___________________________Mg

9. A VW bus's gas tank holds 15.9 gal. Convert this volume to liters.

ANSWER:_______________________________L

10. The apparent speed limit on Highway U.S. 101 is 85 miles/hr. Convert to m/sec.

ANSWER:____________________________m/sec

11. An auto engine has a displacement of 1.6 liters. What is it in cubic centimeters?

ANSWER:______________________________cm^3

12. Mount Fuji is 12,365 feet high. How high is it in m?

ANSWER:______________________________m

13. The world record for the long jump, set at the Tokyo Olympics, is 8.95 m. What is it in feet?

ANSWER:________________________________ft

14. A block has dimensions of 6.0 inches x 3.26 cm x 1.5 feet. What is its volume in mL and in L?

ANSWER:_________________________________L

15. The speed of light is 3.60×10^8 m/sec. How far does light travel per year in miles?

ANSWER:_________________________________

16. Many mechanical pencils use 0.4 millimeter lead. What is the diameter in feet?

ANSWER:________________________________ft

17. For the equation, $PV = nRT$, solve for V and then calculate V using these values. Ignore units.

$P = 2.31$ $n = 0.0272$ $R = 0.082$ $T = 298$

ANSWER:_______________________________=V

18. $$\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2}$$

Solve for P_1 if $V_1 = 429$ $T_1 = 298$

$V_2 = 530$ $T_2 = 201$ $P_2 = 736$

ANSWER:_______________________________$=P_1$

19. Using the same equation, solve for T_2 if

$P_1 = 1.8$ $V_1 = 5.6$ $T_1 = 323$

$P_2 = 6.7$ $V_2 = 8.3$

ANSWER:_______________________________$=T_2$

20. Copper has a density of 8.96 g/cm^3. If a penny weighs 3.05 g, what is its volume in cubic centimeters and in liters?

ANSWER:______________cm^3_____________L